Economic Foundations and Applications of Risk

Part B. Applications

Chapter 8: The Value of Information

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Syllabus

- 8.1 Introduction
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8.1 Introduction

- Since risk comes into existence through the absence of perfect information, and since people tend to dislike risk, **there should be a value to acquiring information.**
- After some **introductory notation** (8.2) . . .
- . . . we analyze the **value of informational signals** (8.3).
- We close by examining a special kind of market failure where individuals disregard their private information and **follow the herd** instead (8.4).

8.2 Some notation

$Pr(s_k | z_i) \neq Pr(z_i | s_k)$

States of the world, signals, and their probabilities

- Let $z_1, z_2, z_3, \cdots$ denote the **states of the world** that realize with an (unconditional) **ex-ante probability** of $p_i = Pr[z_i]$
- Let there be a system of **signals** $s_1, s_2, s_3, \cdots$, with $\pi_k = Pr[s_k]$ denoting the (unconditional) **probability** of s_k realizing.
- Let $Pr[z_i \cap s_k]$ denote the **common probability** of state z_i and signal s_k.
- Let $Pr[s_k \mid z_i] = \frac{Pr[z_i \cap s_k]}{p_i}$ denote the **conditional probability** of signal s_k given state z_i.

Bayes Theorem

- Let us denote the inverse conditional probability, $Pr[z_i \mid s_k] = \frac{Pr[z_i \cap s_k]}{\pi_k}$, as the **ex-post probability** of state z_i conditional on signal s_k having realized.

Refresher: Bayes' theorem

- Remember the definition of conditional probabilities:

$$Pr(B|A) = \frac{Pr(A \cap B)}{Pr(A)}$$

$$Pr[A \mid B] = \frac{Pr[A \cap B]}{Pr[B]},$$

where $(A; B) \subset \Omega^2$ is a pair of events and Ω is the set of all possible events.

- Also recall (with some shudder) **Bayes' theorem**:

$$Pr[B \mid A] = \frac{Pr[A \mid B] \cdot Pr[B]}{Pr[A \mid B] \cdot Pr[B] + Pr[A \mid \overline{B}] \cdot Pr[\overline{B}]} = \frac{Pr[A \mid B] \cdot Pr[B]}{Pr[A]}.$$

$$Pr(z_i | s_k) = \frac{Pr(s_k | z_i) \cdot Pr(z_i)}{Pr(s_k | z_i) \cdot Pr(z_i) + Pr(s_k | \bar{z}_i) \cdot Pr(\bar{z}_i)} = \frac{Pr(s_k | z_i) \cdot Pr(z_i)}{Pr(s_k)}$$

8.3 The value of signals

- Let V denote the **value of the signal system in utility terms:**

$$V = EU^{Signal} - EU^{No\ signal}$$

- About the sign of V:
  - If the signal leads to a change in action, V is positive
  - If the signal does not lead to a change in action, the value of the signal is zero
  - also (quite trivially) if the signal is received after action was already taken, the value of the signal is also zero.

Example: Oil drilling

- Let there be a risk-neutral oil company interested in acquiring land for drilling.
- With equal probability there is oil or there is none ($p_1 = p_2 = \frac{1}{2}$).
- If the company does not drill, its certain payoff will be 0.
- If it chooses to drill and strikes oil, its payoff is 3.
- If it drills and does not strike oil its payoff is $-\frac{3}{2}$.
- Expected profit of oil drilling **if there is no signal:** $\frac{1}{2} \cdot (-\frac{3}{2}) + \frac{1}{2} \cdot 3 = \frac{3}{4} > 0$, so **the company will drill.**

- Now, suppose the company could test-drill for oil, with the following **test technology:** There can be a bad signal (s_1) indicating that there is no oil or a good signal (s_2) indicating that there is oil.
- The **quality of the test** is given by the following matrix of conditional probabilities of test results, given the true state of the world ($Pr[s_k \mid z_i]$):

	z_1 (no oil)	z_2 (oil)
s_1 (signal bad)	$\frac{3}{5}$	$\frac{1}{5}$
s_2 (signal good)	$\frac{2}{5}$	$\frac{4}{5}$
$\sum$	1	1

False negative

False positive

$$Pr(z_i \mid s_k)$$

- From this we can **compute the ex-post probabilities** by using Bayes' theorem:

	z_1 (no oil)	z_2 (oil)	$\sum$
s_1 (signal bad)	$\frac{3}{4}$	$\frac{1}{4}$	1
s_2 (signal good)	$\frac{1}{3}$	$\frac{2}{3}$	1

$$Pr(z_1 \mid s_1) = \frac{Pr(s_1 \mid z_1) \cdot Pr(z_1)}{Pr(s_1 \mid z_1) \cdot Pr(z_1) + Pr(s_1 \mid z_2) \cdot Pr(z_2)}$$

$$= \frac{3/5 \cdot \frac{1}{2}}{3/5 \cdot \frac{1}{2} + \frac{1}{5} \cdot \frac{1}{2}} = \frac{3}{4}$$

$$0.4 =$$

"No Oil"

- **If the company receives the signal** s_1, the expected revenue from drilling is equal to $\frac{3}{4} \cdot (-\frac{3}{2}) + \frac{1}{4} \cdot 3 = -\frac{3}{8} < 0$. So it would be **optimal not to drill** then.
- **If it receives the signal** s_2, expected revenue from drilling would be $\frac{1}{3} \cdot (-\frac{3}{2}) + \frac{2}{3} \cdot 3 = \frac{3}{2} > 0$, and **drilling would be optimal.**
- With probabilities of $Pr[s_1] = 0.4$ and $Pr[s_2] = 0.6$, the **advantage of the signal system,** V, can be calculated as follows:
  - $EU^{No\ signal}$ (Drill): $\frac{1}{2} \cdot (-\frac{3}{2}) + \frac{1}{2} \cdot 3 = \frac{3}{4}$ ← See above
  - EU^{Signal} (Drill if s_2; Don't drill if s_1):

$$\underbrace{0.4}_{Pr(s_1)} \cdot 0 + \underbrace{0.6}_{Pr(s_2)} \cdot \left[\underbrace{\frac{1}{3} \cdot \left(-\frac{3}{2} \right) + \frac{2}{3} \cdot 3}_{3/2} \right] = \frac{9}{10}$$

  - $V = EU^{Signal} - EU^{No\ signal} = \frac{9}{10} - \frac{3}{4} = 0.15$

The Hirshleifer paradox: Is information always valuable?

- Suppose there are two states of the world occurring with probability p and $1-p$, respectively, and k consumers with initial endowments $\overline{x}_{k1}, \overline{x}_{k2}$.
- Now, define EU_k^* as

$$\max_{x_{k1}, x_{k2}} \{pu(x_{k1}) + (1-p)u(x_{k2})\} \equiv EU_k^*$$

$$\text{s.t. } p_1 x_{k1} + p_2 x_{k2} \leq p_1 \overline{x}_{k1} + p_2 \overline{x}_{k2}$$

- Individual rationality implies:

$$EU_k^* \geq \overline{EU}_k \equiv pu(\overline{x}_{k1}) + (1-p)u(\overline{x}_{k2}),$$

i.e. nobody can be made worse off by voluntary trade.

- Now, suppose there is a **public signal before trading** can occur, so that **everybody knows the true state of the world.**
- In that case, **no trade will happen** because nobody would be willing to trade income in the state that everybody knows will occur for income in a state that will not occur.
- Thus, the ex-ante value of the signal is negative.
- However, if there are other ways than free market trade for society to react to the information, then all information may be socially valuable (e.g. flood warnings).
- **Further situations, where additional information is undesired:**
  - Some people may be averse to information for psychological reasons (e.g. genetic testing).
  - Commitment without prior information (E.g.: The problem to prove that you are truly in love with a rich person)

8.4 Herding

Are markets always efficient in aggregating information?

- cf.: Banerjee (QJE, 1992): "Rational herding"
- **Consider the following stylized situation:**
  - There are n agents that have to choose between 2 alternatives A and B (e.g. 2 restaurants).
  - All share an initial belief **(publicly observable signal)** that A is better than B.
  - Each agent receives an **additional private signal** whether A or B is better. These private signals are of identical quality, they are i.i.d. and they are **more informative than the public signal.**
  - The agents have to decide sequentially.
  - The agents can observe whether agents before them have chosen A or B, but cannot observe the signals these agents have received.
- **Question: Will your private signal be helpful** in making your choice?

The signal of the first agent can always be inferred.

- The initial belief is $A \succ B$.
- Either, the first agent receives the affirming private signal $A \succ B$, in which case she will will clearly choose A.
- Or, the first agent receives the contradicting private signal $B \succ A$. As the private signal is more informative than the public signal, the first agent will choose B.
- Hence, we can infer the signal from the choice of the first agent.

Consider the possible scenarios for the subsequent agents.

1. Agent 1 has chosen A.
   - Agent 2 knows that agent 1 must have received signal $A \succ B$.
   - Hence, if agent 2 receives the same signal she will choose A, too.
   - But if she receives signal $B \succ A$ she will also choose A. Why?
   - Her signal is just good enough to exactly offset agent 1's signal. So these two signals cancel out and she has to stick with her initial belief.
   - The same reasoning applies to all subsequent agents. Therefore all following agents will choose A.
   - Rational herding: Note: Only agent 1's action is informative for the entire population. There is **no (efficient) aggregation of information** by the market.

2. Agent 1 has chosen B.
   - Agent 2 knows that agent 1 must have received signal $B \succ A$.
   - Hence, if agent 2 receives the same signal she will choose B, too.
   - From there on, all following agents will also choose B. Why?
   - Each agent can receive at most one signal $A \succ B$ which offsets only 1 of these two 'early' signals. So there is still one excess signal for B left.
   - But if agent 2 receives signal $A \succ B$ she will choose A. Why?
   - Again: Her signal is just good enough to offset agent 1's signal. So these two signals cancel out and she has to stick with the initial belief $A \succ B$.
   - This situation now is the same as the initial one.

- So, with high probability we will end up in a **situation where no additional information is generated because people disregard their private information and follow the herd.**