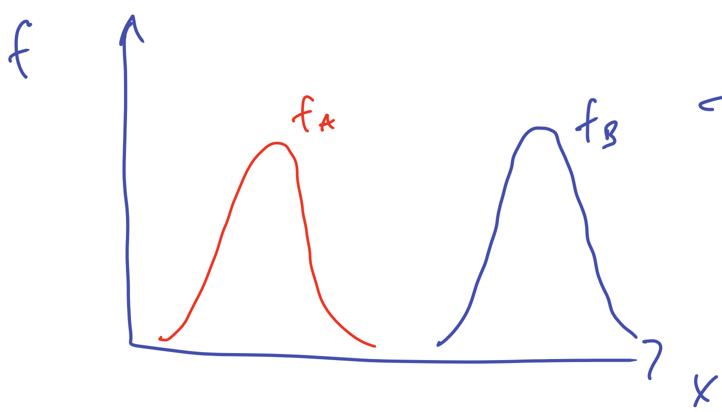
Day 2

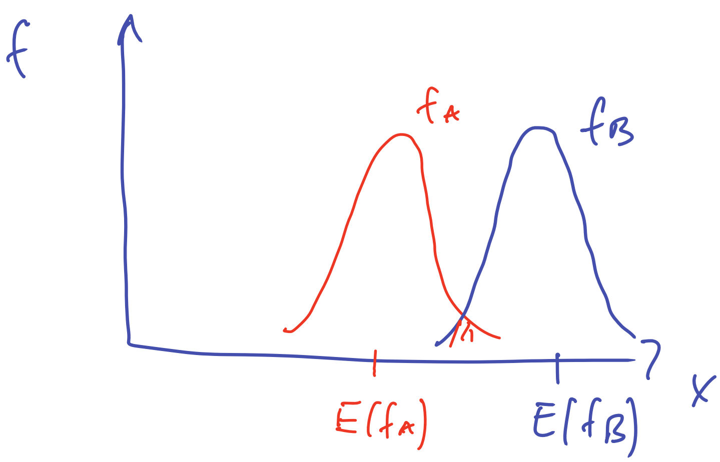
Measures of Risk

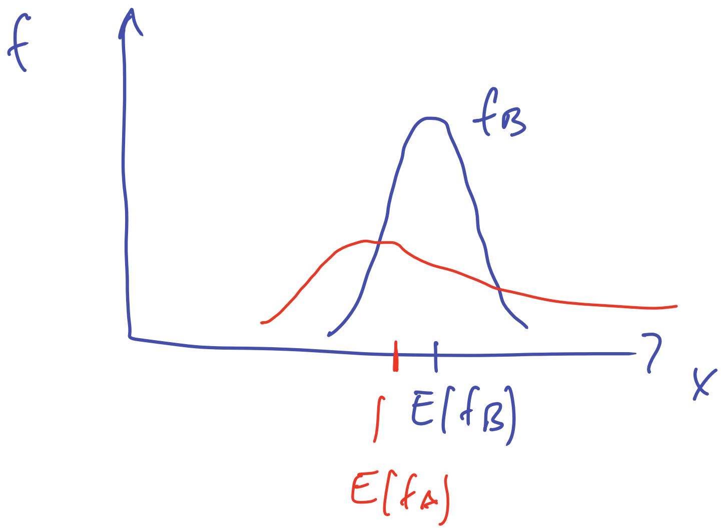
$$L_1 = \left(\tfrac{7}{8}, \tfrac{1}{8} ; 1, 9 \right)$$

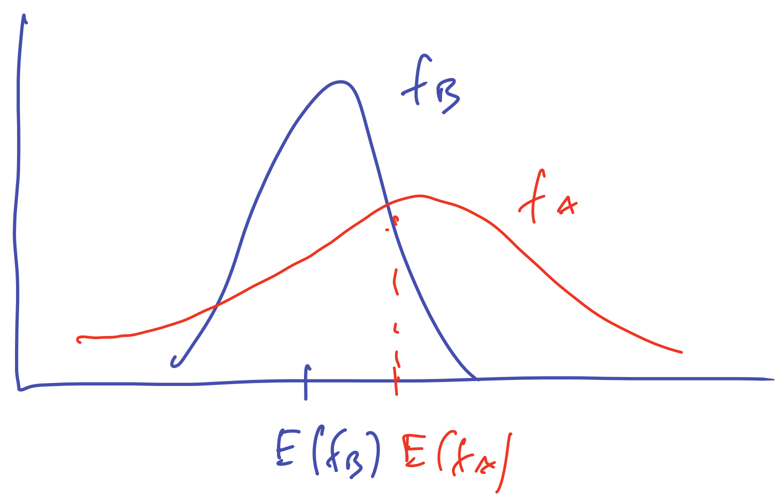
$$L_2 = \left(\tfrac{1}{2}, \tfrac{1}{2} ; 0, 4 \right)$$



Strict Dominance




f
f_B
x
$E(f_B)$
$E(f_A)$


f_B
f_A
$E(f_B)$
$E(f_A)$

$$L_1 = \begin{cases} \frac{7}{8} & 1 \\ \frac{1}{8} & 9 \end{cases} \qquad L_2 = \begin{cases} \frac{1}{2} & 0 \\ \frac{1}{2} & 4 \end{cases}$$

$E(L_1) = 2$ $\qquad$ $E(L_2) = 2$

$Var(L_1) = 7$ $\qquad$ $Var(L_2) = 4$

Example: $\sqrt{x}$

$EU(L_1) = \frac{7}{8}\sqrt{1} + \frac{1}{8}\sqrt{9} = \frac{10}{8}$

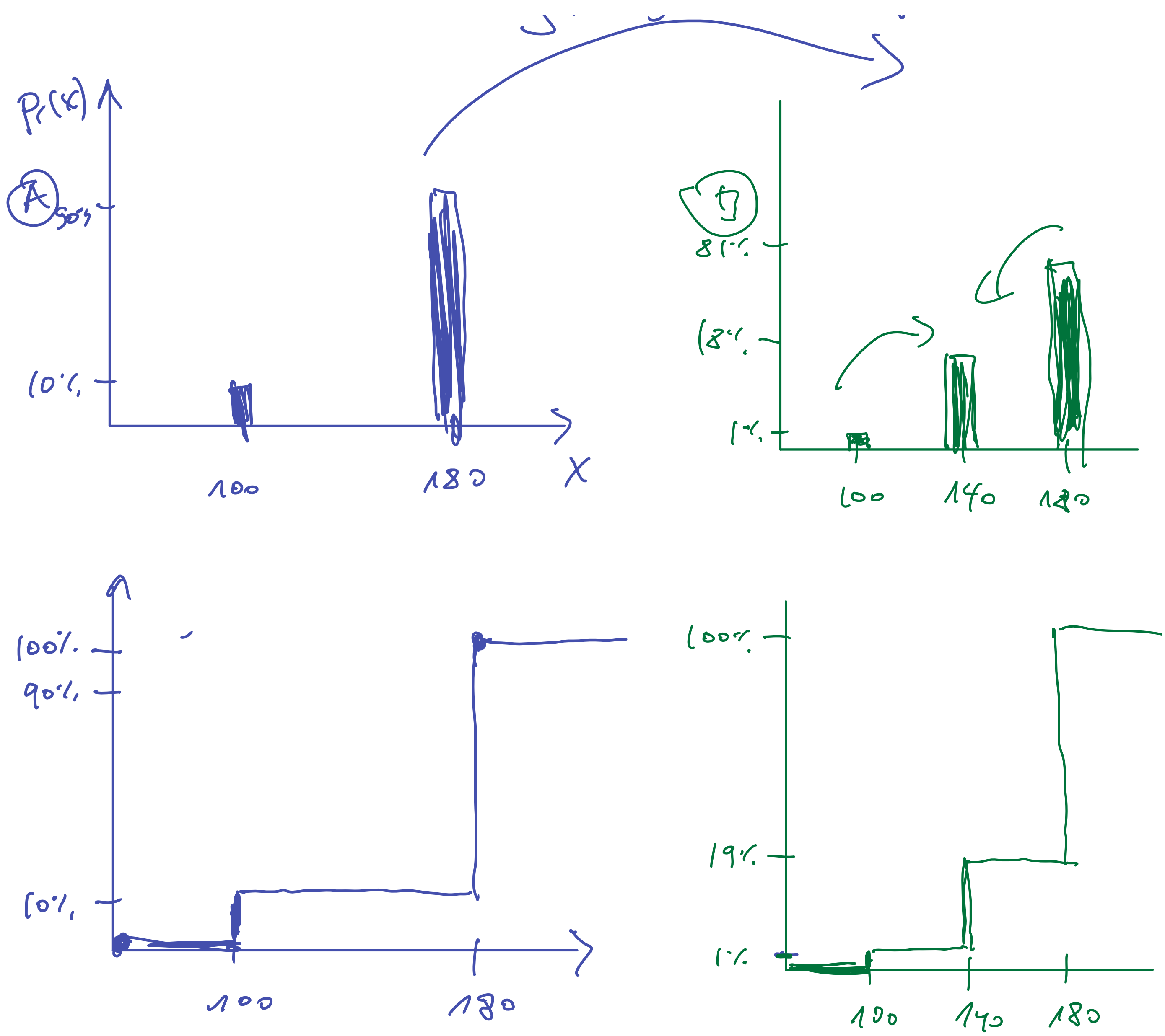
$EU(L_2)$ $\frac{1}{2} \cdot \sqrt{0} + \frac{1}{2}\sqrt{4} = 1$

PS 1.2

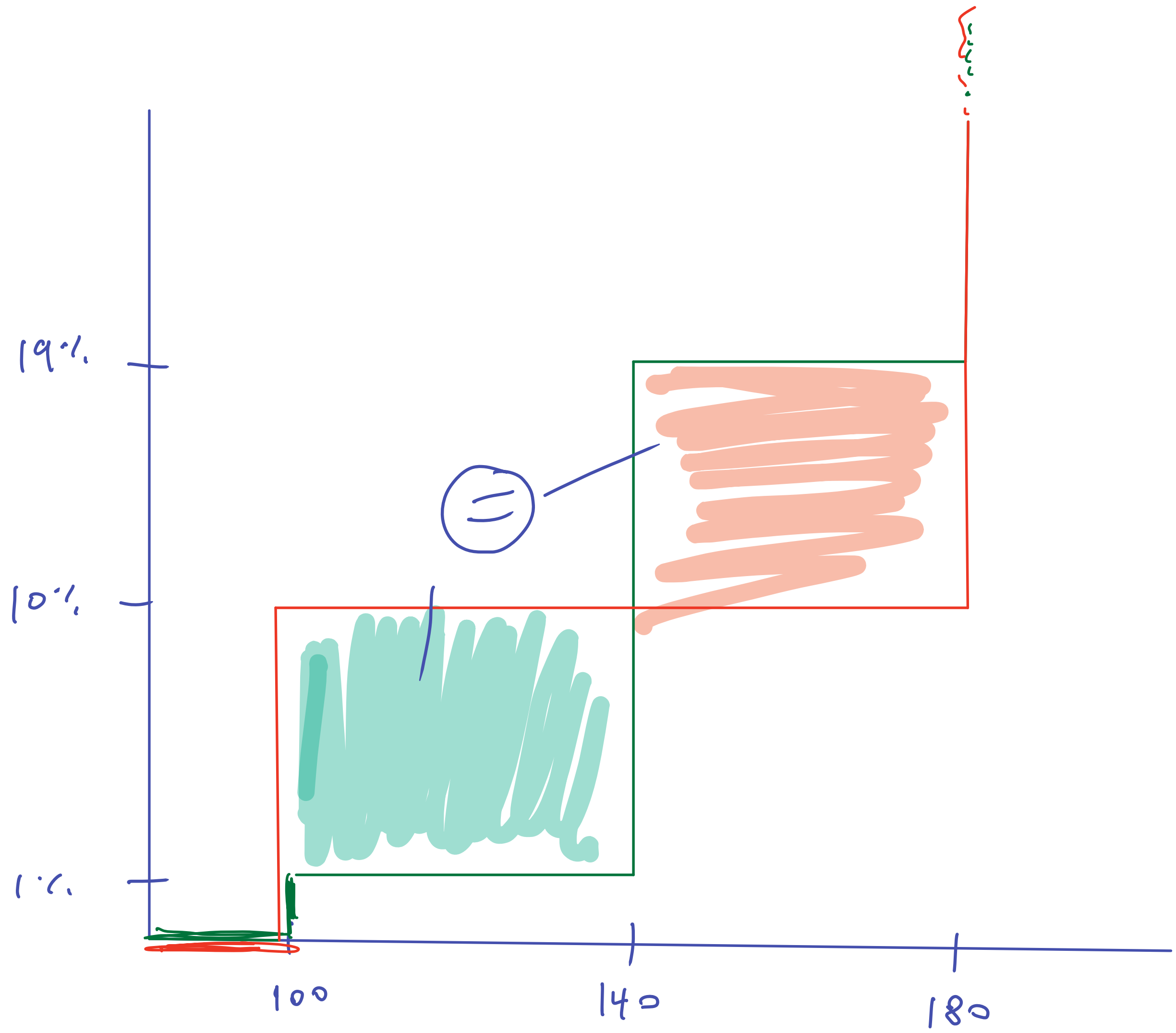
$E(L_A) = 0.9 \cdot 180 + 0.1 \cdot 100 = 172$

$E(L_B) = 0.81 \cdot 180 + 0.18 \cdot 140 + 0.01 \cdot 100 = 172$

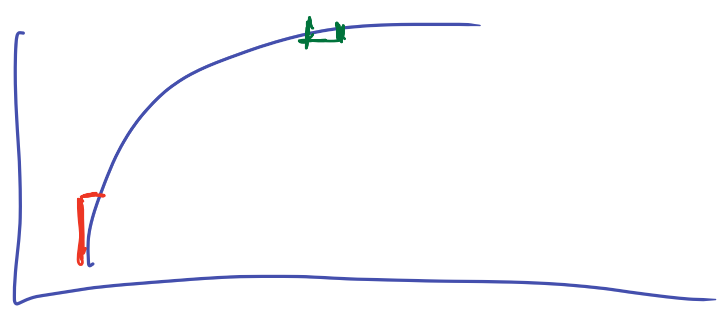
" strong" Decrease of Risk



Pr(x)
A
50%
10%
100
180
X
B
81%
18%
1%
100
140
180
100%
90%
10%
100
180
100%
19%
1%
100
140
180

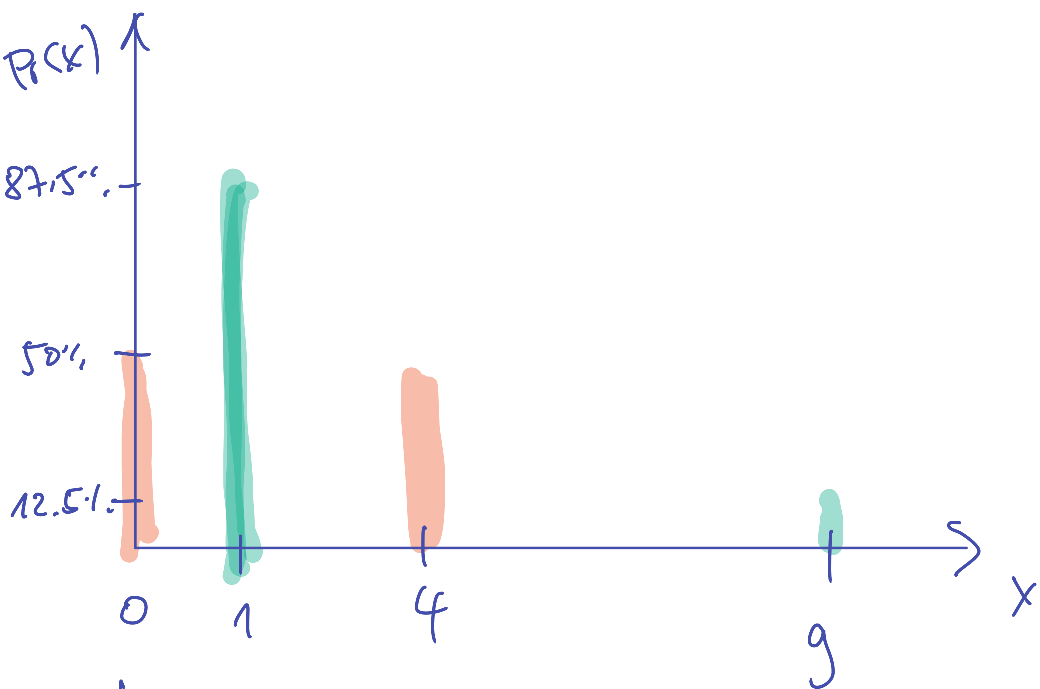


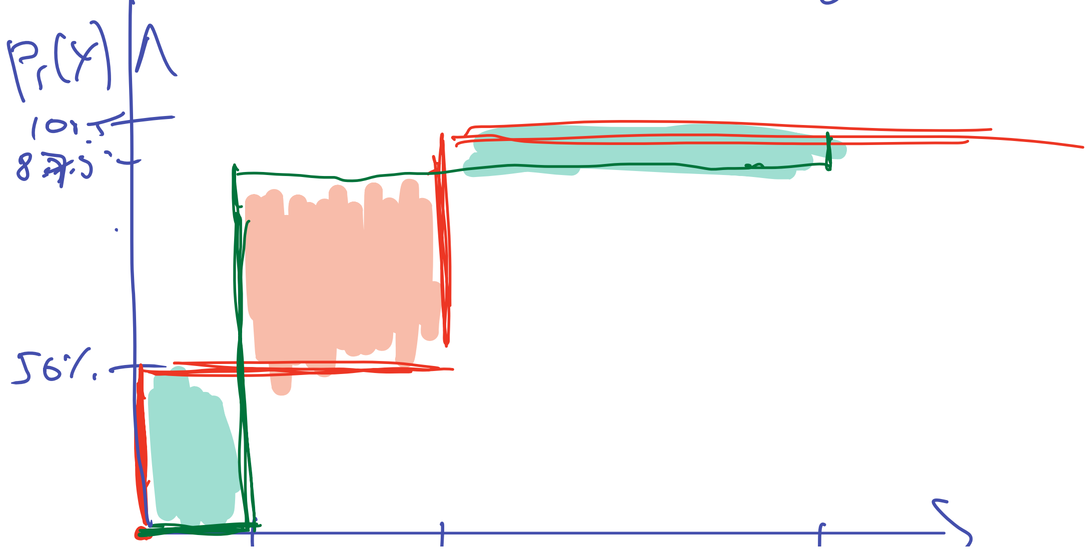
$L_B \succ_{SOSD} L_A$ for risk averse



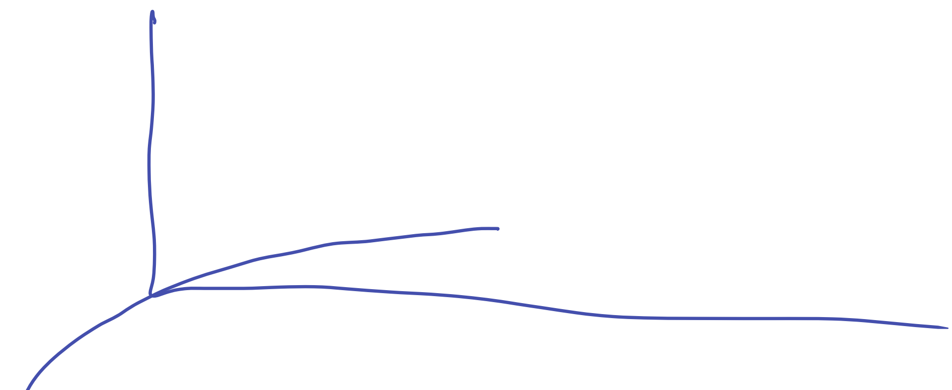
$L_1 = \left(\frac{7}{8} \,\middle|\, \frac{1}{8} ; 1 , 9 \right)$

$L_2 = \left(\frac{1}{2} \,\middle|\, \frac{1}{2} \,\middle|\, 0 , 4 \right)$





1 4 9 x



$$\sqrt{x} \Rightarrow \sqrt{10+x}$$

Refresher: $E[\cdot]$

Linearity

$$E[c\cdot\tilde{X}] = c\cdot E[\tilde{X}]$$

$$E[\tilde{X}+c] = E[\tilde{X}]+c$$

$$E[\tilde{X}+\tilde{Y}] = E(\tilde{X})+E(\tilde{Y})$$

Multiplicativity (?)

$$E(\tilde{X}\cdot\tilde{Y}] = E(\tilde{X})\cdot E(\tilde{Y}) + \underbrace{\mathrm{Cov}(X,Y)}_{=0 \text{ if } \tilde{X} \text{ and } \tilde{Y} \text{ are independent}}$$

$\mathrm{Var}(\cdot)$

$$\mathrm{Var}(\tilde{X}+c) = \mathrm{Var}(\tilde{X})$$

$$\boxed{PS2.1}$$

$$\max_{a,m} EU = p \cdot \ln\left(m(1+i) + a(1+\bar{x})\right) + \ldots$$

$$\ldots + (1-p) \cdot \ln\left(m(1+i) + a(1+\underline{x})\right)$$

$$\underline{\underline{\text{s.t.}}} \quad W \leq a + m$$

$$W = a + m$$

$$\max_{a} EU = p \cdot \ln\left[(W-a)(1+i) + a(1+\bar{x})\right] + \ldots$$

$$\ldots + (1-p) \ln\left((W-a)(1+i) + a(1+\underline{x})\right)$$

$$= p \cdot \ln\left[W(1+i) + a(\bar{x}-i)\right] + (1-p) \ln\left[W(1+i) + a(\underline{x}-i)\right]$$

$$\underline{\underline{\text{FOC:}}} \quad \frac{\partial EU}{\partial a} = \boxed{p \cdot \frac{\bar{x}-i}{W(1+i) + a(\bar{x}-i)} + (1-p) \frac{\underline{x}-i}{W(1+i) + a(\underline{x}-i)} = 0}$$

$\oplus$

E(Marginal Benefit of buying a)

$\ominus$

E(Marginal "Cost" of buying a)

Check Upload on Website

$E(x)$

$$a^* = \frac{W(1+i)(\mu - i)}{(\bar{x} - i)(\underline{x} - i)} > 0$$

$\oplus$ $\ominus$

$\mu > i$

$a(1+x)$

$m(1+i)$

$$\frac{\partial a^*}{\partial W} > 0$$

$$= -\frac{(1+i)(\mu-i)}{(\bar{x}-i)(\underline{x}-i)} > 0$$

↑ DARA

$$\frac{a^*}{W} = -\frac{(1+i)(\mu-i)\cancel{W}}{(\bar{x}-i)(\underline{x}-i)\cancel{W}}$$

$$\frac{\partial\left(\frac{a^*}{w}\right)}{\partial W} = 0$$